# An Alternative Elementary Proof for Fermat's Last Theorem

# P. N. Seetharaman



Abstract: Fermat's Last Theorem states that the equation  $x^n$  +  $y^n = z^n$  has no solution for x, y and z as positive integers, where n is any positive integer > 2. Taking the proofs of Fermat and Euler for the exponents n = 4 and n = 3, it would suffice to prove the theorem for the exponent n = p, where p is any prime > 3. We hypothesize that r, s and t are positive integers satisfying the equation  $r^p + s^p = t^p$  and establish a contradiction in this proof. We include another Auxiliary equation  $x^3 + y^3 = z^3$  and connect these two equations by using transformation equations. On solving the transformation equation we get rst = 0, thus proving that only a trivial solution exists in the main equation  $r^p + s^p = t^p$ .

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## I. INTRODUCTION

Around 1637. Pierre de Fermat French а Mathematician, wrote in the margin of his book, claiming that he has found a marvelous proof for the equation  $x^n + y^n$  $= z^n$ , but the margin was too narrow to contain it. His proof is available only for the equation  $x^4 + y^4 = z^4$ , which he had proved using "infinite descent" method. Later on Euler proved the theorem in the equation  $x^3 + y^3 = z^3$  [1].

Many mathematicians like Dirichlet, Legendre, Gabril Lame proved the theorem for the exponents n = 5 and n = 7. Around 1820, Sophie Germain gave a remarkable proof for  $x^{\ell} + y^{\ell} = z^{\ell}$  where  $\ell$  and  $(2 \ell + 1)$  are both odd primes and  $\ell$  does divide xyz [2]. Ernst Kummer made the first substantial step in proving Fermt's Last theorem for Regular Primes [3]. Many mathematicians worked on this theorem by which number theory developed leaps and bounds [4]. Mathematicians found a close relationship between Fermat's Last theorem and Elliptic curve. Finally in 1995 Andrew proved Wiles the theorem completely. Many mathematicians have analysed and explained the theorem in all aspects. In this proof, we are trying for an alternative elementary proof for Fermat's Last theorem.

#### **II. ASSUMPTIONS**

1) We presume that all r, s and t are non-zero positive integers in the equation  $r^{p} + s^{p} = t^{p}$  where p is any prime > 3, and establish a contradiction. gcd(r,s,t) = 1. Any two of r, s and t cannot simultaneously be squares.

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P.N Seetharaman\*, Retired Executive Engineer, Energy Conservation Cell), Tamil Nadu State Electricity Board, Chennai (Tamil Nadu), India. Email ID: palamadaiseetharaman@gmail.com, ORCID ID: 0000-0002-4615-1280

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- We are using another auxiliary equation  $x^3 + y^3 = z^3$ 2) (already proved) and we connect the above two equations means of transformation equations by using the parameters called a, b, c, d, e and f.
- 3) Since we are proving the theorem only in the equation  $r^{p} + s^{p} = t^{p}$ , we have the choice of assigning numerical values for the equation  $x^3 + y^3 = z^3$ . In this proof we give the values x = 29; y = 71;  $z^3 = 29^3 + 71^3 = 10^2 \times$ 3823 for convenience.
- We have used F, E and R in the transformation 4) equations in which we define E and R as distinct odd primes each coprime to each of x, y,  $z^3$ , r. s and t. F =  $(3823rs)^3$ .
- We may have r, s and t as coprimes to each of 29, 71 5) and 3823; otherwise we have the choice of assigning alternative values such that x = 11; y = 53;  $z^3 = 11^3 +$  $53^3 = 8^2 \times 2347$  such that r, s & t will be coprime to 11, 53 and 2347.

**Proof.** By trials, we have created the following equations

$$\left(a\sqrt{t^{p}}+b\sqrt{F^{1/3}}\right)^{2}+\left(c\sqrt{E^{5/3}}+d\sqrt{3823}\right)^{2}=\left(e\sqrt{71}+f\sqrt{R^{1/3}}\right)^{2}$$

and

$$\left( a\sqrt{F^{5/3}} - b\sqrt{s^p} \right)^2 + \left( c\sqrt{29} - d\sqrt{E^{1/3}} \right)^2 = \\ \left( e\sqrt{R^{5/3}} - f\sqrt{r^p} \right)^2 \dots (1)$$

to be the transformation equations of  $x^3 + y^3 = z^3$  and  $r^p$  +  $s^p = t^p$  respectively through the parameters called a, b, c, d, e and f. Here we have assigned numerical values for x = 29; y = 71;  $z^3 = 29^3 + 71^3 = 10^2 \times 3823$ . *E* and *R* are distinct odd primes and  $F = (3823rs)^3$ . We may have r, s and t as coprimes to 29, 71 and 3823. Otherwise we have the choice of assigning suitable alternative numerical values for x, yand  $z^3$  such that x = 11; y = 53;  $z^3 = 11^3 + 53^3 = 8^2 \times 2347$ and so on such that r, s and t will be coprimes to the new odd primes 11, 53 and 2347.

From equation (1), we get

$$a\sqrt{t^{p}} + b\sqrt{F^{1/3}} = \sqrt{x^{3}} \quad \dots \quad (2)$$

$$a\sqrt{F^{5/3}} - b\sqrt{s^{p}} = \sqrt{r^{p}} \quad \dots \quad (3)$$

$$c\sqrt{E^{5/3}} + d\sqrt{3823} = \sqrt{y^{3}} \quad \dots \quad (4)$$

$$c\sqrt{29} - d\sqrt{E^{1/3}} = \sqrt{s^{p}} \quad \dots \quad (5)$$

$$e\sqrt{71} + f\sqrt{R^{1/3}} = \sqrt{z^{3}} \quad \dots \quad (6)$$
And

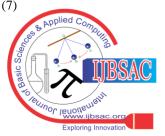
$$e\sqrt{R^{5/3}} - f\sqrt{r^p} = \sqrt{t^p} \quad \dots$$

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## An Alternative Elementary Proof for Fermat's Last Theorem

Solving simultaneously (2) and (3), (4) and (5), (6) and (7), we get

$$a = \left(\sqrt{x^{3}s^{p}} + \sqrt{F^{1/3}r^{p}}\right) / \left(F + \sqrt{s^{p}t^{p}}\right)$$

$$b = \left(\sqrt{F^{5/3}x^{3}} - \sqrt{r^{p}t^{p}}\right) / \left(F + \sqrt{s^{p}t^{p}}\right)$$

$$c = \left(\sqrt{E^{1/3}y^{3}} + \sqrt{3823s^{p}}\right) / \left(E + \sqrt{29 \times 3823}\right)$$

$$d = \left(\sqrt{29y^{3}} - \sqrt{E^{5/3}s^{p}}\right) / \left(E + \sqrt{29 \times 3823}\right)$$

$$e = \left(\sqrt{z^{3}r^{p}} + \sqrt{R^{1/3}t^{p}}\right) / \left(R + \sqrt{71r^{p}}\right)$$
and
$$f = \left(\sqrt{R^{5/3}z^{3}} - \sqrt{71t^{p}}\right) / \left(R + \sqrt{71r^{p}}\right)$$
From (2) & (7), we get
$$\sqrt{t^{p}} \times \sqrt{t^{p}} = \left(\sqrt{x^{3}} - b\sqrt{F^{1/3}}\right) \left(e\sqrt{R^{5/3}} - f\sqrt{r^{p}}\right) / (a)$$
i.e.,
$$t^{p} = \left\{(e)\sqrt{R^{5/3}x^{3}} - (f)\sqrt{r^{p}x^{3}} - (be)\sqrt{F^{1/3}R^{5/3}} + (bf)\sqrt{F^{1/3}r^{p}}\right\} / (a)$$
From (3) & (5), we have

$$\sqrt{r^{p}} \times \sqrt{r^{p}} = \left(a\sqrt{F^{5/3}} - b\sqrt{s^{p}}\right) \left(e\sqrt{R^{5/3}} - d\sqrt{t^{p}}\right) / (f)$$
  
i.e.,  $r^{p} = \left\{(ae)\sqrt{F^{5/3}R^{5/3}} - (ad)\sqrt{F^{5/3}t^{p}} - (be)\sqrt{R^{5/3}s^{p}} + (bd)\sqrt{s^{p}t^{p}}\right\} / (f)$ 

From (3) & (5), we get

$$\sqrt{s^{p}} \times \sqrt{s^{p}} = \left(a\sqrt{F^{5/3}} - \sqrt{r^{p}}\right)\left(c\sqrt{29} - d\sqrt{E^{1/3}}\right) / (b)$$
  
i.e.,  $s^{p} = \left\{(ac)\sqrt{29F^{5/3}} - (ad)\sqrt{F^{5/3}E^{1/3}} - (c)\sqrt{29r^{p}} + (d)\sqrt{E^{1/3}r^{p}}\right\} / (b)$ 

Substituting the equivalent values of  $t^p$ ,  $r^p$  and  $s^p$  in the Fermat's equation  $r^p + s^p = t^p$  after multiplying both sides by  $\{abf\}$ , we get

$$\{bf\} \{(e)\sqrt{R^{5/3}x^3} - (f)\sqrt{x^3r^p} - (be)\sqrt{F^{1/3}R^{5/3}} + (bf)\sqrt{F^{1/3}r^p} \}$$

$$= (ab)\{(ae)\sqrt{F^{5/3}R^{5/3}} - (ad)\sqrt{F^{5/3}t^p} - (be)\sqrt{R^{5/3}s^p} + (bd)\sqrt{s^pt^p} \}$$

$$+ (af)\{(ac)\sqrt{29F^{5/3}} - (ad)\sqrt{F^{5/3}E^{1/3}} - (c)\sqrt{29r^p} + (d)\sqrt{E^{1/3}r^p} \}$$

$$(8)$$

Our aim is to compute all rational terms in equation (8) and equate them on both sides. To facilitate this, let us multiply both sides of equation (8) by

$$\left\{ \left(F + \sqrt{s^{p}t^{p}}\right)^{3} \left(E + \sqrt{29 \times 3823}\right) \left(R + \sqrt{71r^{p}}\right)^{2} \right\}$$

for freeing from denominators on the parameters a, b, c, d, e and f, and again we multiply both sides by  $(\sqrt{3823 \times s})$  for getting some rational terms.

I term in LHS of equation (8), after multiplying by the respective terms and substituting for  $\{b(ef)\}$ 

$$=\sqrt{R^{5/3}x^3}\left(F^2 + s^p t^p + 2F\sqrt{s^p t^p}\right)\left(E + \sqrt{29 \times 3823}\right)$$
$$\left(\sqrt{F^{5/3}x^3} - \sqrt{r^p t^p}\right)\left(\sqrt{z^3 r^p} + \sqrt{R^{1/3} t^p}\right)\sqrt{3823 \times s}\left(\sqrt{R^{5/3} z^3} - \sqrt{71t^p}\right)$$

On multiplying by

$$\left\{\sqrt{R^{5/3}x^3}\left(2F\sqrt{s^pt^p}\right)\sqrt{29\times3823}\left(-\sqrt{r^pt^p}\right)\sqrt{R^{1/3}t^p}\sqrt{3823\times s}\left(-\sqrt{71t^p}\right)\right\}$$

we get

 $\left\{ \left(2FR \times 3823\right) \sqrt{29x^3} \left(t^{2p} \sqrt{s^{p+1}}\right) \sqrt{71r^p} \right\}$ 

which will be irrational, if r is coprime to 71; otherwise we have the choice of assigning

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alternative numerical values of y such that r is coprime to the new value of y.

II term in LHS of equation (8), after multiplying by the respective terms and substituting for  $\{bf^2)\}$ 

$$= \left(-\sqrt{x^{3}r^{p}}\right)\left(F^{2} + s^{p}t^{p} + 2F\sqrt{s^{p}t^{p}}\right)\left(E + \sqrt{29 \times 3823}\right)\sqrt{3823 \times s}$$
$$\left(\sqrt{F^{5/3}x^{3}} - \sqrt{r^{p}t^{p}}\right)\left(R^{5/3}z^{3} + 71t^{p} - 2\sqrt{R^{5/3} \times 71z^{3}t^{p}}\right)$$

On multiplying by

$$\left(-\sqrt{x^{3}r^{p}}\right)\left(2F\sqrt{s^{p}t^{p}}\right)\sqrt{29\times3823}\sqrt{3823\times s}\left(-\sqrt{r^{p}t^{p}}\right)\left(71t^{p}\right)$$
$$\left\{\left(2\times71F\times3823\right)\sqrt{29x^{3}}\left(t^{2p}r^{p}\sqrt{s^{p+1}}\right)\right\}$$

we get

which is rational.

III term in LHS of equation (8), after multiplying by the respective terms and substituting for  $\{b^2(ef)\}\$ 

$$= \left(-\sqrt{F^{1/3}R^{5/3}}\right) \left(F + \sqrt{s^{p}t^{p}}\right) \left(E + \sqrt{29 \times 3823}\right) \sqrt{3823 \times s}$$
$$\left(F^{5/3}x^{3} + r^{p}t^{p} - 2\sqrt{F^{5/3}x^{3}r^{p}t^{p}}\right) \left(\sqrt{z^{3}r^{p}} + \sqrt{R^{1/3}t^{p}}\right) \left(\sqrt{R^{5/3}z^{3}} - \sqrt{71t^{p}}\right)$$

(i) on multiplying by

$$\left\{ \left( -\sqrt{F^{1/3}R^{5/3}} \right) \sqrt{s^{p}t^{p}} \sqrt{29 \times 3823} \sqrt{3823 \times s} \left( -2\sqrt{F^{5/3}x^{3}r^{p}t^{p}} \right) \sqrt{R^{1/3}t^{p}} \left( -\sqrt{71t^{p}} \right) \right\}$$

we get

$$\left\{-\left(2FR\times 3823\right)\sqrt{29x^3}\left(t^{2p}\sqrt{s^{p+1}}\right)\sqrt{71r^p}\right\}$$

which will be irrational. Also this term gets cancelled with the term worked out under I term in LHS, above. (ii) also on multiplying by

$$\left\{ \left( -\sqrt{F^{1/3}R^{5/3}} \right) \sqrt{s^{p}t^{p}} \left( E \right) \sqrt{3823 \times s} \left( r^{p}t^{p} \right) \sqrt{R^{1/3}t^{p}} \left( -\sqrt{71t^{p}} \right) \right\}$$

$$\left\{ \left( ER \right) \left( r^{p}t^{2p}\sqrt{s^{p+1}} \right) \sqrt{F^{1/3} \times 3823 \times 71t^{p}} \right\}$$

we get

which will be irrational since  $F^{1/3} = (3823rs)$  and  $\sqrt{71rst^p}$  will be irrational if *r*, *s* & *t* are coprimes to 71. IV term in LHS of equation (8), after multiplying by the respective terms and substituting for  $\{b^2f^2\}$ 

$$= \sqrt{F^{1/3}r^{p}} \left(F + \sqrt{s^{p}t^{p}}\right) \left(E + \sqrt{29 \times 3823}\right) \sqrt{3823 \times s}$$
$$\left(F^{5/3}x^{3} + r^{p}t^{p} - 2\sqrt{F^{5/3}x^{3}r^{p}t^{p}}\right) \left(R^{5/3}z^{3} + 71t^{p} - 2\sqrt{R^{5/3} \times 71z^{3}t^{p}}\right)$$

(i) on multiplying by

$$\sqrt{F^{1/3}r^{p}}\sqrt{s^{p}t^{p}}\sqrt{29\times 3823}\sqrt{3823\times s}\left(-2\sqrt{F^{5/3}x^{3}r^{p}t^{p}}\right)\left(71t^{p}\right)$$

we get

$$\left\{-\left(2\times71\times3823F\right)\sqrt{29x^3}\left(r^pt^{2p}\sqrt{s^{p+1}}\right)\right\}$$

This rational term gets cancelled with the rational term worked out under II term in LHS above. (ii) also on multiplying by

$$\left\{\sqrt{F^{1/3}r^p}\left(F\times E\right)\sqrt{3823\times s}\left(r^pt^p\right)\left(71t^p\right)\right\}$$

we get

$$\left\{ \left(71FEr^{p}t^{2p}\right)\sqrt{F^{1/3}\times 3823\times r^{p}s}\right\}$$

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which will be rational since  $F^{1/3} = (3823rs)$ .

## I term in RHS of equation (8), after multiplying by the respective terms and substituting for $\{(a^2b)e)\}$

$$= \left(\sqrt{F^{5/3}R^{5/3}}\right) \left(R + \sqrt{s^{p}t^{p}}\right) \left(E + \sqrt{29 \times 3823}\right) \sqrt{3823 \times s}$$
$$\left(x^{3}s^{p} + F^{1/3}r^{p} + 2\sqrt{F^{1/3}x^{3}r^{p}s^{p}}\right) \left(\sqrt{z^{3}r^{p}} + \sqrt{R^{1/3}t^{p}}\right) \left(\sqrt{F^{5/3}x^{3}} - \sqrt{r^{p}t^{p}}\right)$$
lying by

on multiplying by

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# An Alternative Elementary Proof for Fermat's Last Theorem

$$\left\{\sqrt{F^{5/3}R^{5/3}}(R)\sqrt{29\times3823}\sqrt{3823\times s}\left(2\sqrt{F^{1/3}x^3r^ps^p}\right)\sqrt{R^{1/3}t^p}\left(-\sqrt{r^pt^p}\right)\right\}$$
$$\left\{-\left(2FR^2\times3823\right)\sqrt{29x^3}\left(r^pt^p\sqrt{s^{p+1}}\right)\right\}$$

we get

which is rational.

II term in RHS of equation (8), after multiplying by the respective terms and substituting for  $\{a^2bd\}$ 

$$= \left(-\sqrt{F^{5/3}t^{p}}\right) \left(R^{2} + 71r^{p} + 2R\sqrt{71r^{p}}\right) \left(E + \sqrt{29 \times 3823}\right) \sqrt{3823 \times s}$$
$$\left(\sqrt{F^{5/3}x^{3}} - \sqrt{r^{p}t^{p}}\right) \left(x^{3}s^{p} + F^{1/3}r^{p} + 2\sqrt{F^{1/3}x^{3}r^{p}s^{p}}\right) \left(\sqrt{29y^{3}} - \sqrt{E^{5/3}s^{p}}\right)$$

On multiplying by

$$\left(\left(-\sqrt{F^{5/3}t^p}\right)\left(2R\sqrt{71r^p}\right)\sqrt{29\times3823}\sqrt{3823\times s}\left(-\sqrt{r^pt^p}\right)\left(2\sqrt{F^{1/3}x^3r^ps^p}\right)\sqrt{29y^3}\right)$$

we get

 $\left\{ \left(4 \times 29 \times 3823 FR\right) \sqrt{71 y^3} \left(r^p t^p \sqrt{s^{p+1}}\right) \sqrt{r^p x^3} \right\}$ 

which will be irrational, if *r* is coprime to x = 29.

III term in RHS of equation (8), after multiplying by the respective terms and substituting for  $\{(ab^2)e\}$ 

$$= \left(-\sqrt{R^{5/3}s^{p}}\right) \left(R + \sqrt{71r^{p}}\right) \left(E + \sqrt{29 \times 3823}\right) \sqrt{3823 \times s}$$

$$\left(\sqrt{x^{3}s^{p}} + \sqrt{F^{1/3}r^{p}}\right) \left(F^{5/3}x^{3} + r^{p}t^{p} - 2\sqrt{F^{5/3}x^{3}r^{p}t^{p}}\right) \left(\sqrt{z^{3}r^{p}} + \sqrt{R^{1/3}t^{p}}\right)$$

$$\left(\left(\sqrt{z^{5/3}z^{2}}\right) \left(z^{2}\right) \sqrt{z^{2}z^{2}}\right) \left(z^{2}\right) \sqrt{z^{2}z^{2}} \left(z^{2}\right) \sqrt{z^{2}z^{2}} \left(z^{2}\right) \sqrt{z^{2}z^{2}} \left(z^{2}\right) \sqrt{z^{2}z^{2}} \left(z^{2}\right) \sqrt{z^{2}} \left(z^{2}\right) \sqrt{z^{2}}$$

(i) on multiplying by

$$\left\{ \left( -\sqrt{R^{5/3} s^p} \right) (R) \sqrt{29 \times 3823} \sqrt{3823 \times s} \sqrt{F^{1/3} r^p} \left( -2\sqrt{F^{5/3} x^3 r^p t^p} \right) \sqrt{R^{1/3} t^p} \right\}$$
  
given by

we get the rational term given by

$$\left\{ \left(2 \times 3823 F R^2\right) \sqrt{29 x^3} \left(r^p t^p \sqrt{s^{p+1}}\right) \right\}$$

this term get cancelled with the rational term worked out and the I term in the RHS above. (ii) also on multiplying by

$$\left(\left(-\sqrt{R^{5/3}s^p}\right)(ER)\sqrt{3823\times s}\sqrt{F^{1/3}r^p}\left(r^pt^p\right)\sqrt{R^{1/3}t^p}\right)$$

we get

$$\left\{-\left(ER^{2}\right)\left(r^{p}t^{p}\sqrt{s^{p+1}}\right)\sqrt{3823\times F^{1/3}r^{p}t^{p}}\right\}$$

which will be irrational, since  $F^{1/3} = (3823rs)$  and  $\sqrt{st^p}$  will be irrational, with gcd(r,t) = 1 and both s & t can not simultaneously be squares.

IV term in RHS of equation (8), after multiplying by the respective terms and substituting for  $\{ab^2d\}$ 

$$= \sqrt{s^{p}t^{p}} \left( E + \sqrt{29 \times 3823} \right) \left( R^{2} + 71r^{p} + 2R\sqrt{71r^{p}} \right) \sqrt{3823 \times s}$$
$$\left( \sqrt{x^{3}s^{p}} + \sqrt{F^{1/3}r^{p}} \right) \left( F^{5/3}x^{3} + r^{p}t^{p} - 2\sqrt{F^{5/3}x^{3}r^{p}t^{p}} \right) \left( \sqrt{29y^{3}} - \sqrt{E^{5/3}s^{p}} \right)$$

(i) On multiplying by

we get the rational term

$$\left\{\sqrt{s^{p}t^{p}}\sqrt{29\times3823}\left(2R\sqrt{71r^{p}}\right)\sqrt{3823\times s}\sqrt{F^{1/3}r^{p}}\left(-2\sqrt{F^{5/3}x^{3}r^{p}t^{p}}\right)\sqrt{29y^{3}}\right\}$$
given by

 $\left\{-\left(4\times 29\times 3823FR\right)\sqrt{71y^3}\left(r^pt^p\sqrt{s^{p+1}}\right)\sqrt{r^px^3}\right\}$ 

which is irrational. Also this term gets cancelled with II term RHS the above. (ii) also on multiplying by

we get

$$(E)\left(r^{p}t^{p}\sqrt{s^{p+1}}\right)\left(R^{2}+71r^{p}\right)\sqrt{F^{1/3}\times 3823r^{p}t^{p}}\right\}$$

 $\left\{\sqrt{s^{p}t^{p}}\left(E\right)\left(R^{2}+71r^{p}\right)\sqrt{3823\times s}\sqrt{F^{1/3}r^{p}}\left(r^{p}t^{p}\right)\right\}$ 





V term in RHS of equation (8), after multiplying by the respective terms and substituting for  $\{a^2cf\}$ 

$$= \sqrt{29F^{5/3}} \left(F + \sqrt{s^{p}t^{p}}\right) \left(R + \sqrt{71r^{p}}\right) \sqrt{3823 \times s} \left(x^{3}s^{p} + F^{1/3}r^{p} + 2\sqrt{F^{1/3}x^{3}r^{p}s^{p}}\right) \left(\sqrt{E^{1/3}y^{3}} + \sqrt{3823 \times s^{p}}\right) \left(\sqrt{R^{5/3}z^{3}} - \sqrt{71t^{p}}\right)$$

on multiplying by

$$\left\{\sqrt{29F^{5/3}}\sqrt{s^{p}t^{p}}\sqrt{71r^{p}}\sqrt{3823\times s}\left(2\sqrt{F^{1/3}x^{3}r^{p}s^{p}}\right)\sqrt{3823\times s^{p}}\left(-\sqrt{71t^{p}}\right)\right\}$$

we get the rational term given by

$$\left\{-\left(2\times3823\times71Fr^{p}s^{p}t^{p}\sqrt{s^{p+1}}\right)\sqrt{29x^{3}}\right\}$$

VI term in RHS of equation (8), after multiplying by the respective terms and substituting for  $\{a^2df\}$ 

$$= \left(-\sqrt{F^{5/3}E^{1/3}}\right) \left(F + \sqrt{s^{p}t^{p}}\right) \left(R + \sqrt{71r^{p}}\right) \sqrt{3823 \times s} \left(x^{3}s^{p} + F^{1/3}r^{p} + 2\sqrt{F^{1/3}x^{3}r^{p}s^{p}}\right) \left(\sqrt{29y^{3}} - \sqrt{E^{5/3}s^{p}}\right) \left(\sqrt{R^{5/3}z^{3}} - \sqrt{71t^{p}}\right)$$

on multiplying by

$$\left\{ \left( -\sqrt{F^{5/3}E^{1/3}} \right) \sqrt{s^{p}t^{p}} \sqrt{71r^{p}} \sqrt{3823 \times s} \left( 2\sqrt{F^{1/3}x^{3}r^{p}s^{p}} \right) \left( -\sqrt{E^{5/3}s^{p}} \right) \left( -\sqrt{71t^{p}} \right) \right\}$$

we get

$$\left\{-\left(2\times71FEr^{p}s^{p}t^{p}\sqrt{s^{p+1}}\right)\sqrt{3823\times x^{3}}\right\}$$

which will be irrational, since x = 29.

VII term in RHS of equation (8), after multiplying by the respective terms and substituting for  $\{a(cf)\}$ 

$$= \left(-\sqrt{29r^{p}}\right) \left(F^{2} + s^{p}t^{p} + 2F\sqrt{s^{p}t^{p}}\right) \left(R + \sqrt{71r^{p}}\right) \sqrt{3823 \times s} \left(\sqrt{x^{3}s^{p}} + \sqrt{F^{1/3}r^{p}}\right) \left(\sqrt{E^{1/3}y^{3}} + \sqrt{3823 \times s^{p}}\right) \left(\sqrt{R^{5/3}z^{3}} - \sqrt{71t^{p}}\right)$$

on multiplying by

$$\left(-\sqrt{29r^{p}}\right)\left(2F\sqrt{s^{p}t^{p}}\right)\sqrt{71r^{p}}\sqrt{3823\times s}\sqrt{x^{3}s^{p}}\sqrt{3823\times s^{p}}\left(-\sqrt{71t^{p}}\right)\right)$$

we get the rational term given by

$$\left\{ \left(2 \times 71 \times 3823 Fr^{p} s^{p} t^{p} \sqrt{s^{p+1}}\right) \sqrt{29x^{3}} \right\}$$

This term gets cancelled with the rational terms worked out under V term in RHS above. VIII term in RHS of equation (8), after multiplying by the respective terms and substituting for  $\{adf\}$ 

$$= \sqrt{E^{1/3}r^{p}} \left(F^{2} + s^{p}t^{p} + 2F\sqrt{s^{p}t^{p}}\right) \left(R + \sqrt{71r^{p}}\right) \sqrt{3823 \times s}$$
$$\left(\sqrt{x^{3}s^{p}} + \sqrt{F^{1/3}r^{p}}\right) \left(\sqrt{29y^{3}} - \sqrt{E^{5/3}s^{p}}\right) \left(\sqrt{R^{5/3}z^{3}} - \sqrt{71t^{p}}\right)$$

On multiplying by

$$\left\{\sqrt{E^{1/3}r^p}\left(2F\sqrt{s^pt^p}\right)\sqrt{71r^p}\sqrt{3823\times s}\sqrt{F^{1/3}r^p}\left(-\sqrt{E^{5/3}s^p}\right)\left(-\sqrt{71t^p}\right)\right\}$$

we get

$$\left\{ \left( 2 \times 71 F E r^p s^p t^p \right) \sqrt{3823 \times F^{1/3} r^p s} \right\}$$

Which will be rational, since  $F^{1/3} = (3823rs)$ .

Sum of all rational part in LHS of equation (8)

$$= \left\{ \left( 71FEr^{p}t^{2p} \right) \sqrt{F^{1/3} \times 3823 \times r^{p}s} \right\} \quad \text{(vide IV terms)}$$

Sum of all rational part in RHS of equation (8)

$$= \left\{ \left( 2 \times 71 F E r^{p} s^{p} t^{p} \right) \sqrt{3823 \times F^{1/3} r^{p} s} \right\} \text{ (vide VIII term)}$$

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Equating the rational terms on both sides of equation (8), we get

$$(71FE)(r^{p}t^{p})\sqrt{3823 \times F^{1/3}r^{p}s}(t^{p}-2s^{p}) = 0$$
$$(71FE)(t^{p}-2s^{p})$$

Dividing both sides by

we get

$$(r^{p}t^{p})\sqrt{3823 \times F^{1/3}r^{p}s} = 0$$
  
i.e.,  $(r^{p}t^{p})(3823 \times s)\sqrt{r^{p+1}} = 0$  (::  $F^{1/3} = 3823s$ 

That is, either r = 0; or s = 0; or t = 0.

This contradicts our hypothesis that all r, s and t are nonzero integers in the equation  $r^p + s^p = t^p$ , where p is any primes > 3, thus proving that only a trivial solution exists in the equation.

## **III. CONCLUSION**

Equation (8) was derived from the two transformation equations by substituting the equivalent values of  $r^p$ ,  $s^p \& t^p$ , in the Fermat's equation  $r^p + s^p = t^p$ . The only main hypothesis that we make in the prrof, namely r, s and t are non-zero integers has been shattered by the result rst = 0, that we proving the theorem.

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#### **AUTHOR'S PROFILE**



P.N. Seetharaman, B.Sc (Mathematics); B.E (Electrical Engineering) is a retired Executive Engineer from Tamil Nadu Electricity Board. He had served in Mettur Tunnel Hydro Power Station for ten years, and finally worked in Research and Development wing, Energy Conservation Cell, at Chennai. He retired from service in 2002.After retirement, he studied Number Theory, especially Fermat's Last Theorem and worked for finding and elementary proof for the Theorem.

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